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CLARIFICACIÓN BASADA EN LA FUNCIÓN DE PERTENENCIA INVERSA Y EL VECTOR UNITARIO

José de Jesús Medel-Juárez[1], Jesús Gómez-Sánchez[1], Rosaura Palma-Orozco[2]

[1]Centro de Investigación en Computación. CIC-IPN. Dpto. Ciencias de la Computación. Avda. Juan de Dios Bátiz, s/n – CP.07738 México, D. F. Tfno.: +52 55 5729 6000. jjmedelj@yahoo.com.mx
[2]Escuela Superior de Cómputo. ESCOM-IPN. Dpto. Posgrado. Avda. Juan de Dios Bátiz, s/n – CP.07738 México, D. F.

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CLARIFICATION BASED ON THE INVERSE MEMBERSHIP FUNCTION AND UNIT VECTOR

ABSTRACT:
This paper presents a clarification model on fuzzy sense, considering that the membership function is invertible with respect to system state viewed as a black-box and defining as an identification with respect to bounded response. In this case was necessary the unit vector based on the membership function values. Specifically, considers the absolute values formed by the difference between the state to be clarified and its average instead of the triangle inequality, allowing the clarification according to the defined membership function. In MatLab® observes the clarification results converging to the reference signal in all most all points describing illustratively how develops the process; and comparing their with traditional centroid method. Now, in the functionals sequence errors converge to a region, that decrements to zero trough the time.

Key Words: *fuzzy logic, clarification, membership functions, computational methods, mean square error.*

RESUMEN:
Este artículo presenta un modelo de clarificación en el sentido borroso, considerando que la función de pertenencia es invertible con respecto al estado del sistema visto como caja negra y definiéndola como proceso de identificación sobre su respuesta acotada. En este caso fue necesaria la descripción del vector unitario basado en los valores de la función de pertenencia. Específicamente, se considera que está formado por diferencia de los valores absolutos entre el valor del estado y su media, en vez de la desigualdad del triángulo; permitiendo así la obtención del identificador dado que se conoce la función de pertenencia. Dentro de las simulaciones en Matlab® se observa que la convergencia entre la señal deseada y la clarificada se da en casi todos los puntos, describiendo ilustrativamente cómo se va desarrollando este proceso; comparándose los resultados con los obtenidos por el método del centroide. De forma tal que en la gráfica en la que se sobreponen los funcionales de error, se observa como convergen a una región que decrece a cero a través del tiempo.

Palabras Clave: *lógica borrosa, clarificación, funciones de pertenencia, métodos computacionales, error cuadrático medio.*

1. INTRODUCTION

The Black-box system with output state observables need to know its internal dynamics and has an effective approach when it uses the fuzzy modeling and identification techniques [1]. In fuzzy logic, the clarification process requires fuzzy modeling operating in ranges established with names known as linguistic variables (LV) [2]. There are several ways to assign values for membership functions through linguistic variables, e.g. classification of random variables in ranges accomplishes the density function. The membership functions commonly used in practice are triangular, trapezoidal, Bell, Gaussian and sigmoid type. The clarification theory and its sensitivity of fuzzy controller commonly use different membership functions. Due to its simple shape and computational efficiency, triangular membership function allows real-time implementations. However, since the membership functions are composed of straight-line segments, they are not smooth in the transition points. On the other hand, Zadeh introduced the extension principle, where the arithmetic of fuzzy numbers gained theoretical and practical importance, permitting the transformation to be a precise measurement [3]. The first clarification technique was the gravity center or centroid, and corresponded to x -coordinate, called value clarified (VC). There are different methods available to clarify, among which are: Adaptive Integration (AI), Blurred

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Media (MB), Semi-Linear Clarification (SLC), Central Area (CA), and Gravity Center (GC), among others. The clarification allows accurate measurement in relation to membership function, according to a linguistic variable (LV), in terms of membership fuzzy set interpreting it in degrees into a specific decision with respect to the actual value. Clarification transforms the fuzzy sets A into significant elements associated with the Black-box system states region $F^{-1}: F_{\tilde{A}}(X) \to X$. Similarly, the clarification is a membership function transformation (fuzzy sets) to significant elements viewed as $F_{\mu}^{-1}: \{\mu_{\tilde{A}}(x) \mid \tilde{A} \in F(X), x \in X\} \to X$, [4]. It selects into a fuzzy set the highest membership ignoring the remaining results, converting a fuzzy element into a specific number that could be a number into the real set. The problem with this type of transformation is the information loss, resulting in many cases the stability loss into logic controllers. This provokes autonomous vehicle accidents, loss or excess pressure in petroleum products, braking abruptly in passenger trains, among other examples. For these reasons, many researchers began to develop methods with better clarification [5]. The purpose of this paper is to present the clarification process transformation based on the concept of unit vector associating the membership function slope to the belonging degree. The simulation results described the clarification advantages regarding traditional results reported in literature about the centroid method. Therefore, to obtain the Black-box system state using clarification it is necessary to know the membership fuzzy degree variable, indicating its occurrence and frequency rate. The results obtained compare and verify the centroid method. In all cases, the centroid always remains constant and does not have any value convergence to the reference system, which is the opposite of what happens with the proposed method.

2. FUZZY SYSTEM CLARIFIED STATES

The most common analytical function corresponds to a triangular membership description [4-6]. According to [7, 8], it is simpler than the Gaussian. Unlike that developed in [9-10], it performs the clarification process of the triangular membership function according to (1), using the unit vector property and slope function (2). Considering u_i^τ entry and exit rate x_i^τ as a Black-box system that in reality meets $\{u_i^\tau\} \subseteq N(\mu_u, \sigma_u^2 < \infty)$ and $\{x_i^\tau\} \subseteq N(\mu_x, \sigma_x^2 < \infty)$, where i is the index of the sequence and τ is the time state evolution of x, with $i \in Z_+, \tau \in R_+$.

The triangular membership function $\mu_{i_{tria}}^\tau$ within a given fuzzy system (1) satisfies the condition $|x_i^\tau - \mu_\tau| < \sigma_\tau$, where μ_τ, σ_τ are the mean and standard deviation respectively. Both with τ time occurrence in the states sequence $\{x_{i\ tria}^\tau\}$ take the membership function $\mu_{i\ tria}^\tau$ where i is the sequence index states occurrence, and τ is the time evolution.

$$\mu_{i\ tria}^\tau = \left\{1 - \frac{|x_i^\tau - \mu_\tau|}{\sigma_\tau} \quad si\ |x_i^\tau - \mu_\tau| < \sigma_\tau; \quad 0 \quad si\ |x_i^\tau - \mu_\tau| \geq \sigma_\tau \right. . \tag{1}$$

With the membership function of an existing Black-box system, the problem arises when (1) needs to clarify the x_i^τ, i.e., the identification system uses a fuzzy variable located in the membership function. Therefore, to find the partial derivative of the membership function $\mu_{i\ tria}^\tau$ regarding$(x_i^\tau - \mu_\tau)$, considers the ownership of the unit vector having $|x_i^\tau - \mu_\tau| = (x_i^\tau - \mu_\tau) sign(x_i^\tau - \mu_\tau)$. However, the derivative of the membership function represented by the same function derives $(x_i^\tau - \mu_\tau)$ having the slope $m(\mu_{i\ tria}^\tau)$, produced in (2).

$$sign\left(m(\mu_{i\ tria}^\tau)\right) = -\frac{sign(x_{i\ tria}^\tau - \mu_\tau)}{\sigma_\tau}. \tag{2}$$

So that, the state clarified $\hat{x}_{i\ tria}^\tau$, as described in (3).

$$\hat{x}_{i\ tria}^\tau = \mu_\tau + \frac{(\mu_{i\ tria}^\tau - 1)}{sign\left(m(\mu_{i\ tria}^\tau)\right)}. \tag{3}$$

The functional error $J_n^q := \boldsymbol{E}\{e_i^{\tau 2}\}_{q=\overline{1,m}}$, $i=\overline{1,n}$, $n,m \in \mathbb{Z}_+$, q represents the sequence number in the sample reference system will be obtained; to find out the region of convergence of the sequence considered; describing in almost every point reference signal x_i^τ, based on the rule of clarification $\hat{x}_i^\tau$, where $e_i^\tau := x_i^\tau - \hat{x}_i^\tau$ is the error of clarification. Recursively $J_n = (1/n)\left(e_n^{\tau 2} - (n-1)J_{n-1}\right)$ considers as stationary conditions [11].

3. RESULTS

This section x_i^τ, response, the black box type system as shown in Figure 1, bounded by a membership function with its first two moments fulfilling stationary conditions to a sequence with $i = 100$, and presents the $\{u_i^\tau\} \subseteq N(\mu_u, \sigma_u^2 < \infty)$.

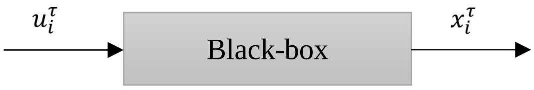

Fig. 1: Black-box system diagram in which display only entry u_i^τ and output x_i^τ.

The information of the stochastic system viewed as a black-box is stored in a Knowledge Base (KB) maintaining the i occurrence rate and the region within the membership function $m(x_i^\tau)$ that will occupy the state x_i^τ, as shown in Figure 2.

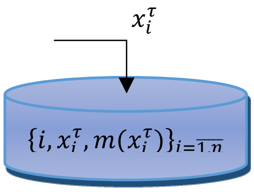

Fig. 2: Scheme of the variables that make up the Knowledge Base (KB) described based on $\{i, x_i^\tau, m(x_i^\tau)\}_{i=\overline{1,n}}$.

In the traditional case, the KB has information based on the relative frequency and the specific membership function. So that, has a basic information [6]. Based on the membership function, the clarified state corresponds to black box weighted response, according to (4).

$$\hat{x}_{i\ c}^\tau = \frac{\sum_{i=1}^k x_i^\tau \mu_{i\ tria}^\tau}{\sum_{i=1}^k \mu_{i\ tria}^\tau} \qquad (4)$$

Figure 3 presents the simulation results clarification (3) and the membership functions elected are triangular $\mu_{i\ tria}^\tau$. The system response has the shape shown in Figure 3 a), and the membership function in Figure 3.b). In accordance with (1), taking the black-box evolution system, the state clarified $\hat{x}_{i\ tria}^\tau$ with respect to observable state x_i^τ, includes the centroid (4). Figure 3.c), presents the clarified obtained $\hat{x}_{i_{tria}}^\tau$ and, in Figure 3.d) the functional error $J_n^q := \boldsymbol{E}\{e_i^{\tau 2}\}_q$, $i=\overline{1,n}$, $n \in \mathbb{Z}_+$, $q=\overline{1,m,}$, representing q the sequence number in the sample reference system, and i the index evolution. If the system was run only once, you have $q = 1$, if twice, you have two q's, one corresponding to $q = 1$ and the other $q = 2$. In this case only has $q = 1$ observed into functional error in Figure 3.e) converging to the reference signal in almost all points.

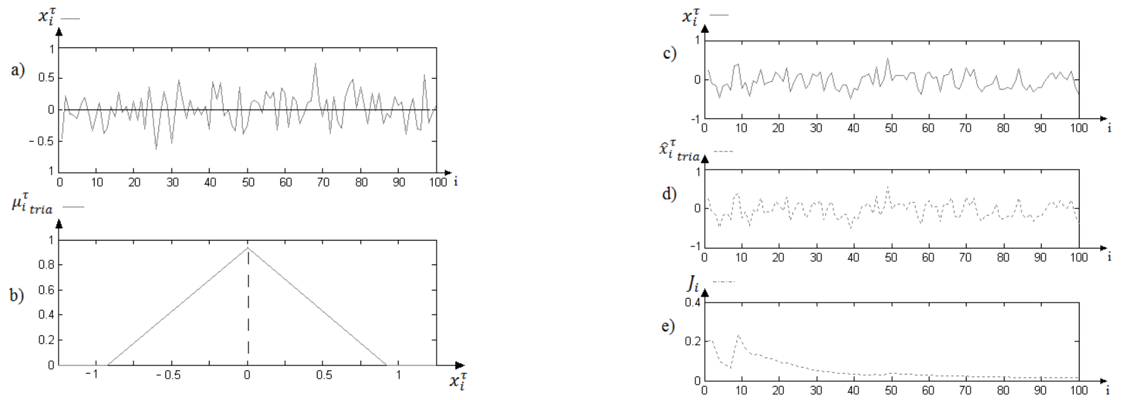

Fig. 3: a) Black-box output signal $\{x_i^\tau : i = \overline{1,n}\}_{n=100}$, *b) Triangular membership function* $\mu_{i\ tria}^\tau\big|_{i=100}$ *with respect to* $\{x_i^\tau\}$, *c) Observable black-box signal* $\{x_i^\tau : i = \overline{1,n}, n \in \mathbb{Z}_+\}_{n=100}$, *d) Clarification state* $\hat{x}_{i\ tria}^\tau$, *e) Functional error* J_i, $q = 1$.

We presents five examples using the triangular membership function with different initial conditions and evolutions. The simulations show stationary smooth conditions changing the first two moments in each graph. From Figures 4 to 8 illustrate the clarification and convergence respect to the reference signals. Figure 9 describes the functional error set finding a common convergence region, which diminishing to zero when the number of iterations tends to infinity.

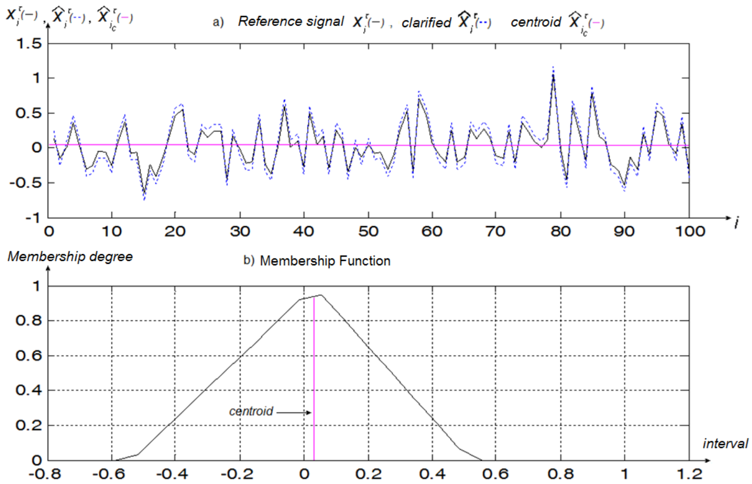

Fig. 4: a) Black-box output signal (x_i^τ) *and clarified signal* ($\hat{x}_{i\ tria}^\tau$) *according to (3) given the triangular membership function* ($\mu_{i\ tria}^\tau$) *with respect to the Knowledge Base (KB) and centroid* ($\hat{x}_{i\ c}^\tau$) *building in accordance with (4) relative to the reference signal* (x_i^τ) *and the membership function* ($\mu_{i\ tria}^\tau$); *b) triangular membership function where the centroid* ($\hat{x}_{i\ c}^\tau$) *projects within a specific range.*

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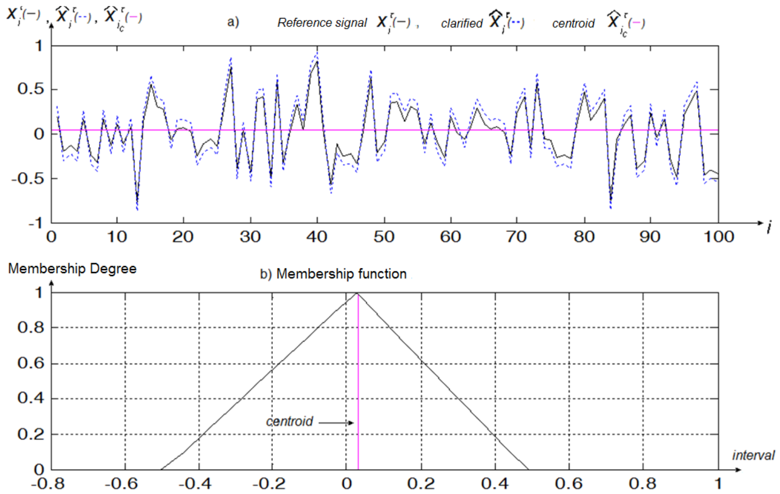

Fig. 5: *a) Black-box output signal (x_i^τ) and clarified signal ($\hat{x}_{i\ tria}^{\tau}$) according to (3) given the triangular membership function ($\mu_{i\ tria}^{\tau}$) with respect to the Knowledge Base (KB) and centroid ($\hat{x}_{i\ c}^{\tau}$) building in accordance with (4) relative to the reference signal (x_i^τ) and the membership function ($\mu_{i\ tria}^{\tau}$); b) triangular membership function where the centroid ($\hat{x}_{i\ c}^{\tau}$) projects within a specific range.*

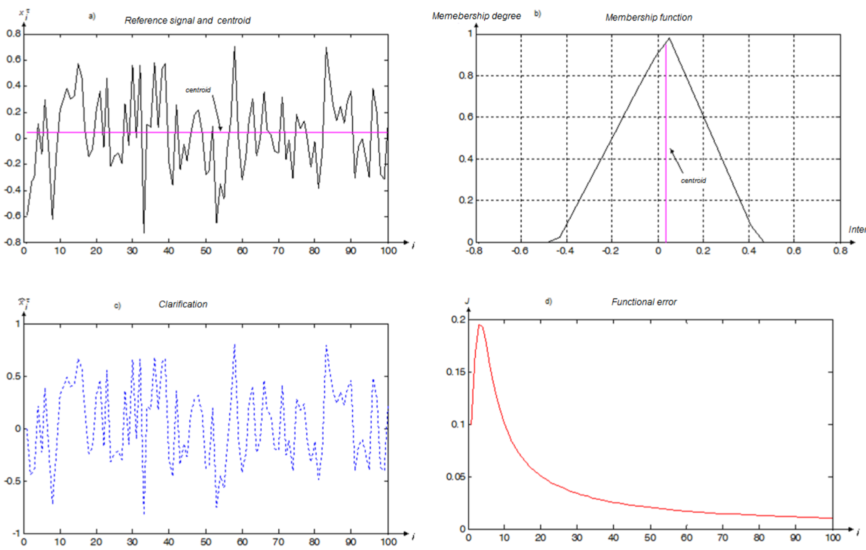

Fig. 6: *a) Black-box output signal (x_i^τ) and centroid ($\hat{x}_{i\ c}^{\tau}$) constructed according to (4); b) Triangular membership function ($\mu_{i\ tria}^{\tau}$) with respect to the Knowledge Base (KB) and centroid ($\hat{x}_{i\ c}^{\tau}$); c) Clarification ($\hat{x}_{i\ tria}^{\tau}$) according to (3) given the triangular membership function ($\mu_{i\ tria}^{\tau}$); d) Functional error $J_{i=\overline{1,100}}^{q=1}$.*

In Figure 7, includes comparison with clarification by the centroid method (4) (pink) regarding the result of the clarification, also reflected in the membership function.

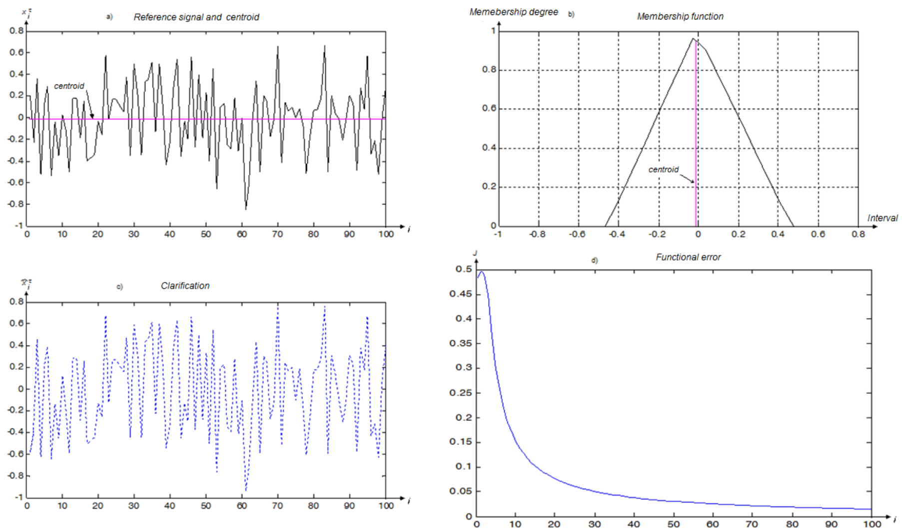

Fig. 7: a) Black-box output signal (x_i^τ) and centroid ($\hat{x}_{i\ c}^\tau$) according to (4); b) Triangular membership function ($\mu_{i\ tria}^\tau$) with respect to the Knowledge Base (KB) and centroid ($\hat{x}_{i\ c}^\tau$); c) Clarification state ($\hat{x}_{i\ tria}^\tau$) according to (3) given the triangular membership function ($\mu_{i\ tria}^\tau$); d) Functional error $J_{i=\overline{1,100}}^{q=1}$.

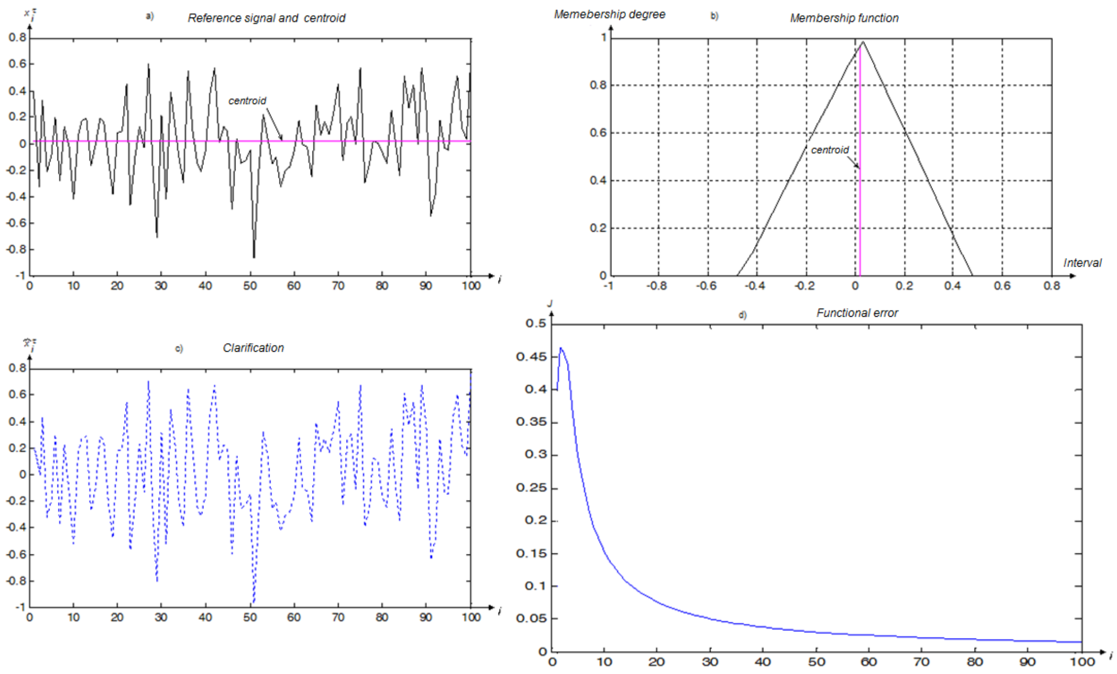

Fig. 8: a) Black-box output signal (x_i^τ) and centroid ($\hat{x}_{i\ c}^\tau$) constructed according to (4); b) triangular membership function ($\mu_{i\ tria}^\tau$) with respect to the Knowledge Base (KB) and centroid ($\hat{x}_{i\ c}^\tau$); c) Clarification state ($\hat{x}_{i\ tria}^\tau$) according to (3) given the triangular membership function ($\mu_{i\ tria}^\tau$); d) Functional error $J_{i=\overline{1,100}}^{q=1}$.

In Figure 9 presents, the functional error set superimposed.

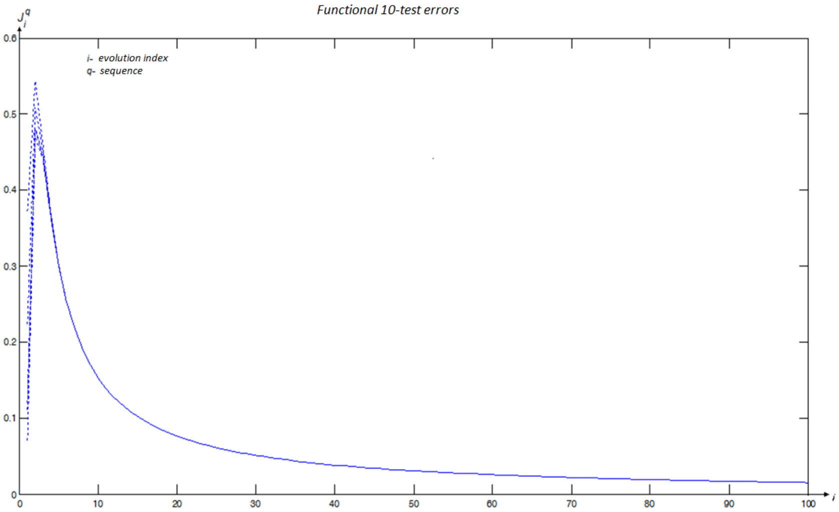

Fig. 9: Functional 10-test errors $(q = 10)$ *made with the clarification process (3) considering* $i = [1,100]$.

Performing the clarification test with pseudo-random variables sequences and their membership functions, having a small bias among them; therefore, the convergence functional error regions shown in Figure 9, generate an excellent expectancy, because the representative convergence region is common to them.

An example of the usefulness and application of (3) is with reference to the incident of the unmanned space mission NASA MESSENGER, launched towards Mercury August 3, 2004 and entered into orbit around that planet on March 18, 2011, starting an orbital period observation during a terrestrial year, as shown in Figure 10.a). Within the theory of digital filtering, considered the Kalman filter as identifier with estimation with essential tools describing the trajectory with specific conditions. So that, on average, the results are not the best, since the explorer would follow the path without considering the surface rough. Consequently, what did happen when the explorer came with that profile landing on mountains? Struck because on average should land on the smooth surface, but the planet has rough surface, shown in Figure 10.b).

The explorer trajectory has the profile shown in Figure 10 c). In the lower solid line is the result of the average landing path and leading to the explorer to an impact region, condition reported as successful without pay attention to the real problem. The dotted line adjacent describes how would it descends based on the projected results. In contrast to a process of setting parameters in the model of the path, a little twist is done in the control surface explorer, tangentially touch causing the edges and can land, as shown by the full continuum top line. In this case, in fuzzy sense, we propose selecting the upper bound of the membership function, rather than the average or lower, making a landing with the least possible impact and thus know the different atmospheric variables, metals and various chemical elements found in the surface of Mercury. Now, instead of thinking about how to salvage what was left of the explorer, could obtain the samples recognizing that this applications required decision based on membership function (1), obtaining clarification (i.e., in this example, the description path) as (3) and verifying its imprecision as the functional error with respect to the rough reference planet surface.

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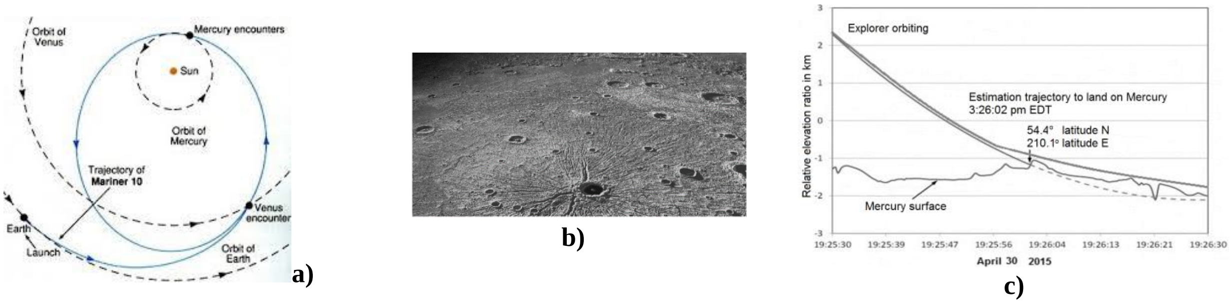

Fig. 10: a) Initial trajectory explorer (NASA, JPL, Applied Physics Laboratory), b) Image of the surface of Mars (NASA, JPL, Applied Physics Laboratory), c) Trajectory landing on the planet's surface Mercury (NASA, JPL, Applied Physics Laboratory) given by continuous bottom line and the proposal on the full continuum top line, using the clarification described by (3).

4. CONCLUSIONS

Clarification is an identification process in the fuzzy sense that reconstructs, describes or predicts the black-box system state with input and output disturbances. Process commonly associated with the centroid method, with maximum dispersion medium limits and maximum amplitude of the reference signal, finding only a single value instead of all sequence states. In spite of, we proposed to build the membership function inverse description. Using as the first function to perform the inverse of triangular function (1). Why this description is used? Because it is a simple and easy feature to implement. Now, we used, the sign function and properties described by the unit vector (2), in where the clarification is possible (3). Hence, the analytical clarification is possible using the inverse function, in particular, the triangular membership function (1), based on the unit vector concept with respect to x_i^τ. The clarification was necessary to keep within an associated function. Where, the order in which the black-box system output response appears x_i^τ indicated with the index τ in time evolution, representing the Knowledge Base (KB) and the slope associated with its state. Thus, having a structure characterized by the time with respect to convergence rate.

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